Multiobjective Optimum Design in Mixed Integer and Discrete Design Variable Problems

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A min-max variant of the global criterion approach is proposed to obtain solutions to multiobjective optimum design problems involving a mix of continuous, discrete, and integer design variables. This modified global criterion approach is used in conjunction with a branch-and-bound algorithm, where the latter was configured especially to accommodate a mix of integer and discrete design variables. The use of a weighting strategy with the proposed method allows the generation of a set of Pareto optimal (noninferior) solutions for both convex and nonconvex problems. The solution strategy is applied to structural design problems involving vector design functions.

Introduction

FORMAL mathematical methods of optimization have found increased applications in a multitude of engineering design problems. In particular, the nonlinear programming approach and the optimality criteria method have been established as flexible and robust solution strategies for such design problems.[1,2] Decisions in engineering design typically require allocation of resources to satisfy multiple, and frequently conflicting, requirements. Despite the recognized multicriterion nature of most design problems, the bulk of research effort has been expended in developing efficient optimization methods for scalar objective function problems. In such an approach, one criterion is selected as the objective function, and the tradeoff among the remaining criterion is resolved by formulating appropriate design constraints. The apparent simplicity afforded by this method is attractive, but an effective case can be made against the use of such an approach. One can argue in favor of a method that deals with multiple criteria, stipulating a "natural" separation of criteria and constraints in any design problem. Furthermore, a treatment of constraints as criteria provides a systematic approach to learn about the extent of the feasible set. In other words, when a multicriterion optimum is obtained, a tradeoff pattern emerges, wherein no criterion may be improved without adversely affecting another. Furthermore, it is also known that the treatment of a criterion as constraints does not yield the same optimum design as would be obtained when solving the optimum design problem as one possessing multiple objectives.

Multicriteria programming has emerged as a subject of special interest in mathematical nonlinear programming. One of the earliest efforts in this area may be attributed to an Italian economist Pareto, who, in 1896, introduced the concept within the framework of welfare economics.[3] The ramifications of this work in optimization theory, operations research, and control theory were recognized only in the late 1960s. Applications of multicriteria optimization in engineering design have also been recognized. Baier[4,5] examined structural design problems in which weight and total energy in several loading conditions are considered as the design criteria. Pareto optimal designs of truss structures have been examined by Koski.[6,7] Numerous examples of design of mechanical structures have been presented, and works of Osyczka[8,9] and Rao and Hati[10] are typical of such applications. Another interesting application of a multiobjective design problem in viscoelasticity is presented in Ref. 11. In addition to several component design problems in aerospace engineering, multiobjective design can prove invaluable in a truly multidisciplinary design environment afforded by the problem of airplane preliminary design. It does appear, however, that the full potential of multicriteria optimization has not been exploited in engineering design.

The present paper proposes an approach for multicriteria design that is derived from a global criterion approach[12] and is especially designed for problems where the design space could consist of continuous, discrete, or even integer design variables. A prerequisite to obtaining a solution to this problem is the availability of a methodology for solving mixed integer discrete optimization problems with a scalar objective function. Such problems have been approached frequently by considering all design variables as continuous, obtaining the optimum solution, and then rounding the specific variables, either up or down, to the nearest integer or discrete point. This simple rounding procedure often fails completely, resulting in either a suboptimal design or, in some cases, even generating an infeasible design.[13]

Early efforts in obtaining a systematic solution to the integer linear programming problem are available in Ref. 14. The branch-and-bound algorithms that emerged later were based on enumeration of space of all feasible integer solutions. The general framework for solving an integer programming problem involves decomposing the original problem into subproblems, modifying constraints to enlarge feasible domains, and finally a process referred to as fathoming. The latter involves checking a solution for feasibility and establishing optimality. The basic strategy used in this work for solving the nonlinear mixed integer programming problems is a variant of the approach proposed by Garfinkel and Nemhauser.[15] The strategy consists of a systematic search of continuous solutions in which the discrete and integer variables are forced successively to assume specific values. The logical structure of the set of solutions was constructed as a binary tree. A modified feasible directions algorithm[16] was used in the solution of the continuous nonlinear programming problem, with piecewise linear representation of the objective function and con-

Received July 22, 1988; revision received March 23, 1989; presented as Paper 89-1217 at the 30th AIAA/ASME/ASCE/AHS Structures, Structural Dynamics, and Materials Conference, Mobile, AL, April 3-5, 1989.

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straints. Details of this approach and a more detailed review of methods in this discipline are presented in Ref. 17.

Subsequent sections of this paper discuss the mathematical statement of the optimization problem. The optimization methodology is discussed with emphasis on formal proof of applicability, and the approach is implemented for purposes of concept verification. Optimum design results obtained for illustrative problems are also presented.

Multicriteria Optimum Design

The strategy used for multicriteria optimization in the present work can be classified broadly as belonging to a category of solution methods with no articulation of preference. In such an approach, also referred to as a global criterion method,[12] a metric function is formulated to represent the distance between the ideal solution and the optimum solution, and a minimization of this function results in the true optimum. For a problem involving m criterion functions, the ideal solution is an m-dimensional vector, the components of which are the optimum values of the individual criteria. These optimum values are obtained by considering each of the criteria separately in a scalar optimization problem. The application of the global criterion method is best illustrated by the sketch in Fig. 1, which shows feasible and infeasible space for a two-criterion function. If the ideal solution were also feasible, there would be no additional effort required. However, this ideal solution is typically infeasible, and a feasible design closest to the ideal solution is sought. The Pareto optimal solution so obtained is such that the design variable vector cannot be altered without adverse effects on any one of the candidate criteria.

One can define a vector objective function $f(x)$ dependent on the design variable vector x, where

$$f(x) = [f_1(x), f_2(x), \ldots, f_i(x), \ldots, f_k(x)]^T \tag{1}$$

$$X = [x_1, x_2, \ldots, x_n]^T$$

and $f_i(X)$ is the ith criterion function. If $f_i^{\mathrm{id}}(x)$ is the ideal solution corresponding to the ith criterion function, the optimal solution is obtained by minimizing a global criterion function of the following form:

$$\text{Minimize } d_\alpha = \left[\sum_{i=1}^{k} |f_i(x) - f_i^{\mathrm{id}}(x)|^\alpha \right]^{1/\alpha} \tag{2}$$

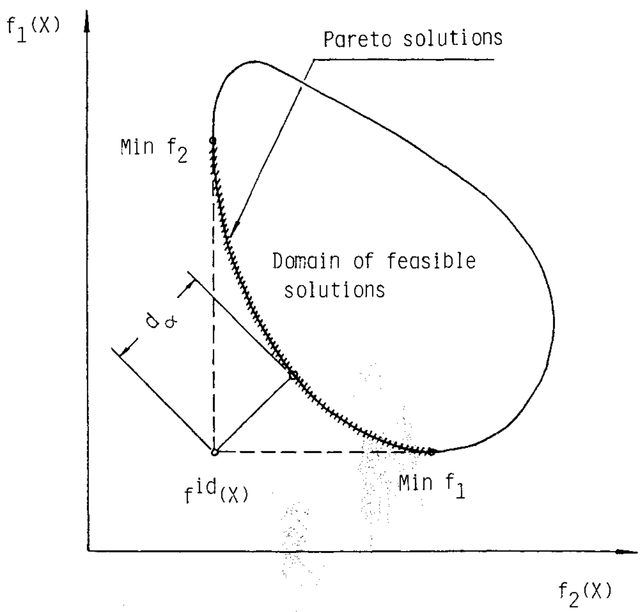

Fig. 1 Graphical representation of Pareto (noninferior) solutions in a problem where two criteria must be minimized.

subject to the prescribed design constraints

$$g_j(x) \le 0, \qquad j = 1,2,\ldots,r \tag{3}$$

$$h_k(x) = 0, \qquad k = r+1,\ldots,m \tag{4}$$

Here d_α is a distance metric with a $1 \le \alpha \le \infty$. Typical choices of $\alpha = 1$ or 2 have been used in the literature.[18,19] Numerical computations are better conditioned if a normalized objective function vector $\tilde{f}(x)$ is used, where

$$\tilde{f}(x) = \left[\tilde{f}_1(x), \tilde{f}_2(x), \ldots \tilde{f}_k(x) \right]^T \tag{5}$$

and the individual components of this vector function are obtained as follows:

$$\tilde{f}_i(x) = \frac{f_i(x) - \min f_i(x)}{\max f_i(x) - \min f_i(x)}, \qquad i = 1,2,\ldots,k \tag{6}$$

In the present approach, a relative deviation metric is preferred over the absolute deviation represented by Eq. (2). The metric function to be minimized is stated as

$$\tilde{f}_i(x) = \frac{f_i(x) - f_i^{\mathrm{id}}(x)}{f_i^{\mathrm{id}}(x)}, \qquad i = 1,2,\ldots,k \tag{7}$$

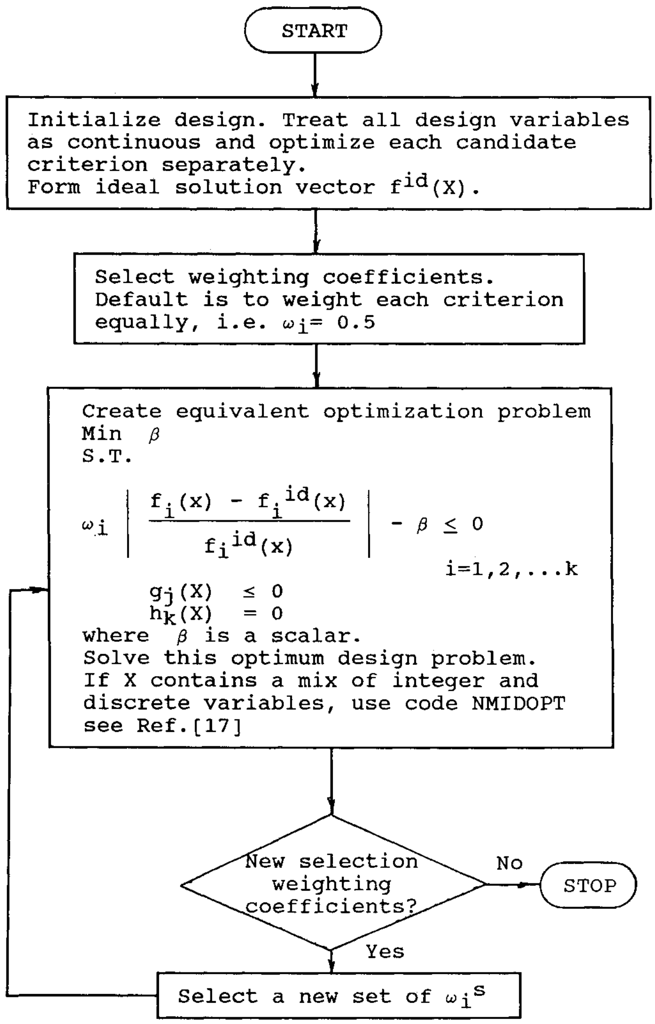

Fig. 2 Flow diagram representation of multiobjective optimization combined with code NMIDOPT for mixed integer and discrete design variable problem.

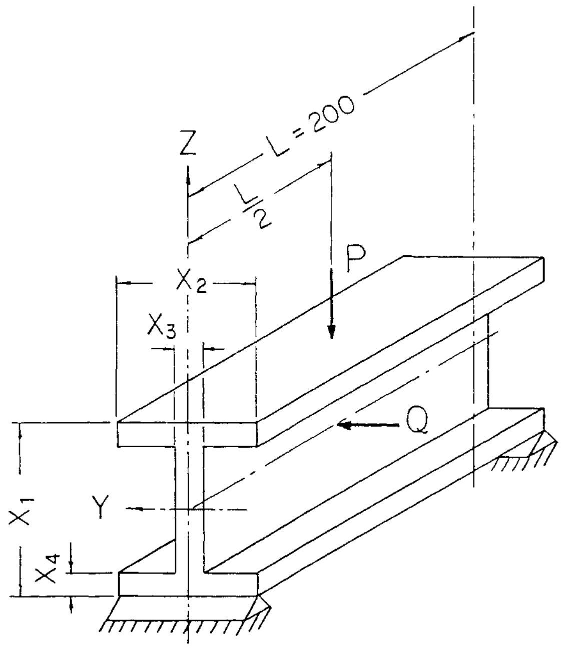

Fig. 3 Definition of design variables and loading for simply supported I-beam.

and the summation in the metric function can be written as

$$d_\alpha = \left[\sum_{i=1}^{k} \left| \frac{f_i(x) - f_i^{\mathrm{id}}(x)}{f_i^{\mathrm{id}}(x)} \right|^\alpha \right]^{1/\alpha} \tag{8}$$

which in the limit of $\alpha \to \infty$ reduces to the following form:

$$d_\alpha = \max_i \left| \frac{f_i(x) - f_i^{\mathrm{id}}(x)}{f_i^{\mathrm{id}}(x)} \right|, \qquad i = 1,2,\ldots,k \tag{9}$$

Minimization of the metric function results in a commonly encountered min-max problem in multiobjective optimization. Therefore, the mathematical statement of the optimization problem may be written as follows:

$$\min_x \max_i \left| \frac{f_i(x) - f_i^{\mathrm{id}}(x)}{f_i^{\mathrm{id}}(x)} \right|, \qquad i = 1,2,\ldots,k \tag{10}$$

The solution to the preceding optimization problem yields the best compromise solution, in which all criteria are considered equally important. Use of weighting coefficients can be introduced in conjunction with this method to rank the importance of the candidate criterion, and the min-max problem is restated as follows:

$$\min_x \max_i \omega_i \left| \frac{f_i(x) - f_i^{\mathrm{id}}(x)}{f_i^{\mathrm{id}}(x)} \right|, \qquad i = 1,2,\ldots,k \tag{11}$$

where ω_i is the weighting coefficient representing the relative importance of the ith criterion.

In a situation where the design variable set is a mixture of continuous, integer, and discrete design variables, the choice of an ideal solution for use in the global criterion method is an important consideration. In the present work, the ideal solution was selected as the one obtained by treating all design variables as continuous. The mathematical basis for this selection is as follows.

First, a scalar variable, β, is introduced to facilitate the transformation of the min-max problem of Eq. (10) into an equivalent scalar optimization problem.

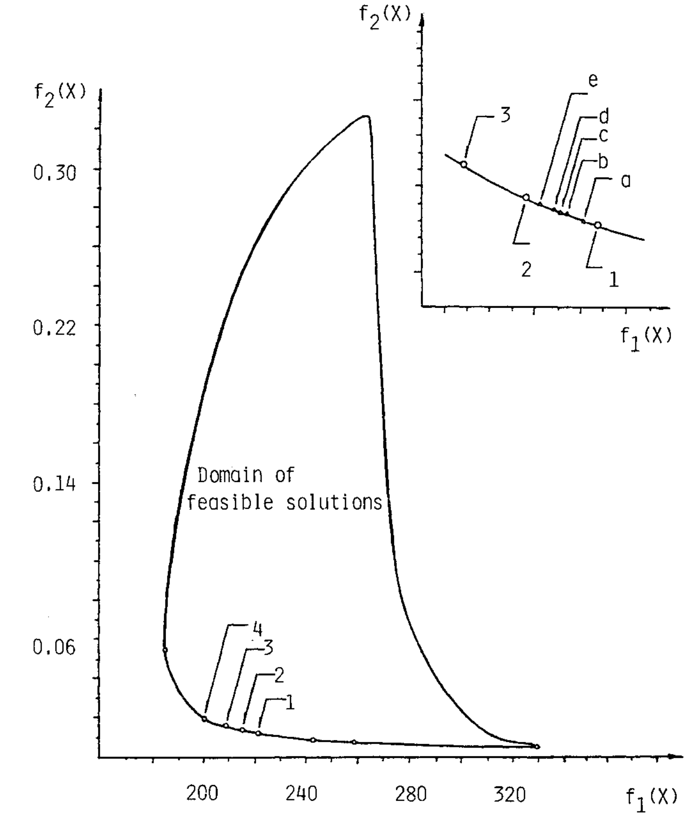

Fig. 4 Graphical representation of the Pareto set of solutions for I-beam problem.

$$\text{Minimize } \beta \tag{12}$$

subject to the following additional constraints:

$$\omega_i \left| \frac{f_i(x) - f_i^{\mathrm{id}}(x)}{f_i^{\mathrm{id}}(x)} \right| - \beta \leq 0, \qquad i = 1,2,\ldots,k \tag{13}$$

Since the first term on the left-hand side of Eq. (13) is always positive, the smallest value of β would be obtained in a situation when the equality is strictly satisfied.

$$\left| \frac{f_i(x)}{f_i^{\mathrm{id}}(x)} - 1 \right| = \frac{\beta}{\omega_i}, \qquad i = 1,2,\ldots,k \tag{14}$$

It can be argued further that the ideal solution obtained by minimizing $f_i(x)$ with all design variables treated as continuous will always be smaller or equal to the function value obtained by imposing discrete or integer requirements on some design variables. Hence,

$$\frac{f_i(x)}{f_i^{\mathrm{id}}(x)} \geq 1 \tag{15}$$

and Eq. (14) can be arranged as follows:

$$f_i(x) = f_i^{\mathrm{id}}(x)\ [1 + (\beta/\omega_i)] \tag{16}$$

From Eq. (16) one can conclude that a smaller value of $f_i^{\mathrm{id}}(x)$ will minimize the function $f_i(x)$ when the scalar variable β is minimized. Therefore, the ideal solution should be obtained by treating all design variables as continuous.

Illustrative Examples

A flowchart for the algorithm described in the preceding section is shown in Fig. 2. The algorithm was first implemented in the design of a simply supported beam,[9] as shown in Fig. 3. The objective of the design problem was to select the design variables x_1, x_2, x_3, and x_4 to minimize both the weight of the beam and its deflection at the midspan under the applied loads. The mathematical statement of the optimum design problem can be written as follows:

$$\text{Minimize } (f_1, f_2)$$

where

$$f_1(x) = 2x_2x_4 + x_3(x_1 - 2x_4) \tag{17}$$

$$f_2(x) = PL^3/48EI \tag{18}$$

$$I = \{x_3(x_1 - 2x_4)^3 + 2x_2x_4\,[4x_4^2 + 3x_1(x_1 - 2x_4)]\}/12 \tag{19}$$

subject to the strength constraints:

$$\frac{M_y}{Z_y} + \frac{M_z}{Z_z} \le \sigma_b \tag{20}$$

Here $P = 600$ kN; $L = 200$ cm; $E = 2 \times 10^4$ kN/cm²; $\sigma_b = 16$ kN/cm² is the permissible bending stress; M_y and M_z are the maximal bending moments in the y and z directions, respectively; and Z_y and Z_z are the section moduli. Additional side constraints are also imposed in the design problem.

$$10 \le x_1 \le 80, \qquad 10 \le x_2 \le 50 \tag{21a}$$

$$0.9 \le x_3 \le 5, \qquad 0.9 \le x_4 \le 5 \tag{21b}$$

A modified feasible usable search direction technique was used to obtain the separately attainable minimum of each component of the objective function as follows:

$$f_1(x_{(1)}^{\text{id}}) = 127.443, \qquad f_2(x_{(1)}^{\text{id}}) = 0.05934$$

$$(x_{(1)}^{\text{id}}) = (61.78,\ 40.81,\ 0.9,\ 0.9)$$

$$f_1(x_{(2)}^{\text{id}}) = 850.0, \qquad f_2(x_{(2)}^{\text{id}}) = 0.005903$$

$$(x_{(2)}^{\text{id}}) = 80.0,\ 50.0,\ 5.0,\ 5.0)$$

Hence the ideal solution vector can be written as

$$f^{(\text{id})} = (127.443,\ 0.005903)$$

Table 1 Representative subset of Pareto optimal (noninferior) solutions for I-beam design problem ($\Sigma\omega_i = 1$)

Case No.	Weighting coefficient (ω_1, ω_2)	Design variables $X = (x_1, x_2, x_3, x_4)$	Objective function $f(X) = (f_1, f_2)$
1	(0.45,0.55)	(79.99,49.99,0.9,2.39)	(307.53,0.0127)
2	(0.55,0.45)	(80.0,50.0,0.9,2.083)	(276.55,0.0143)
3	(0.65,0.35)	(79.99,50.0,0.9,1.791)	(247.88,0.0163)
4	(0.08,0.20)	(80.0,39.79,0.9,1.725)	(206.14,0.0205)

Table 2 Optimum results for I-beam problem for three different variable types (weighting coefficient $\omega_i = 0.5$ used).

Case description $X = (x_1, x_2, x_3, x_4)$	Objective function $f(X) = (f_1, f_2)$
All real continuous variables $X = (79.99,49.99,0.9,2.235)$	(291.43,0.01351)
x_3 and x_4 are discrete type $X = (80.0,48.69,0.9,2.30)$	(291.85,0.01350)
x_3 is integer type; x_4 discrete type $X = (80.0,49.72,1,2.20)$	(294.38,0.01362)

Using a weighting coefficient strategy, one can obtain a set of Pareto solutions, as illustrated in Fig. 4. All points in the interior of the feasible space represent inferior solutions, because one can always find a point on the boundary for which both criteria can be improved simultaneously. Points on the boundary, however, belong to a noninferior, or Pareto set. A subset of these Pareto optimal solutions is presented in Table 1. If more alternatives are required between points 1 and 2, the weighting coefficients are adjusted to assume values between 0.45 and 0.55. Representative alternative solutions are indicated by points a–e in Fig. 4.

The problem was then modified to introduce discrete andinteger design variables. Variables x_3 and x_4 were chosen to be of the discrete type, assuming values between 0.9 and 5.0, in

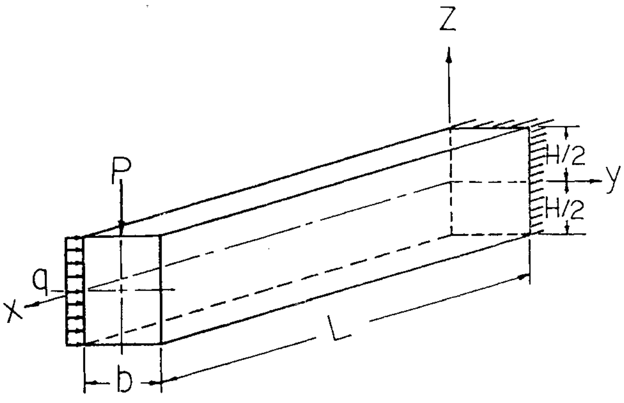

Fig. 5 Configuration and loading of composite laminated beam.

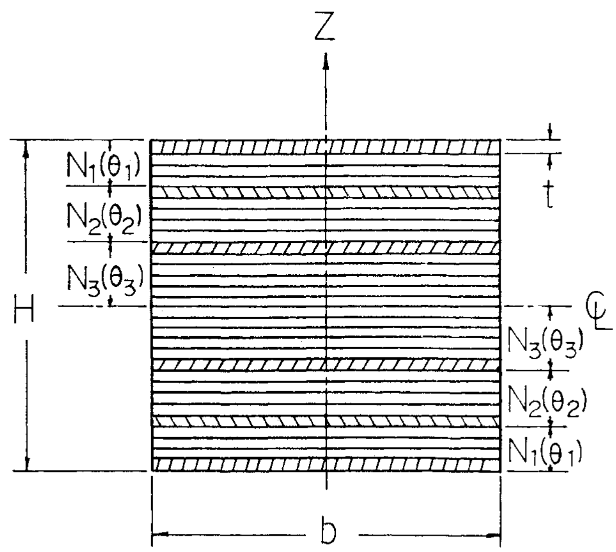

Fig. 6 Geometrical configuration of multilayered laminated structure.

Table 3 Set of Pareto optimal solutions for Scotch composite structure

Weight coefficient	Weight, N	z deflection, mm	Variable type
(0.30,0.70)	2.3698	0.02582	All continuous
(0.30,0.70)	2.4365	0.02674	Mixed
(0.50,0.50)	2.1276	0.03584	All continuous
(0.50,0.50)	2.139	0.03585	Mixed
(0.70,0.30)	1.8322	0.05602	All continuous
(0.70,0.30)	1.889	0.05177	Mixed

Table 4 Multiobjective optimum results of mixed integer and discrete design variables for two different composite structures

	Final optimum design									
Fiber	t, mm	N_1	N_2	N_3	θ_1, deg	θ_2, deg	θ_3, deg	V_f	F, N	Δ_z, mm
Scotch	0.225	6	24	21	−8.9	6.1	−11.7	0.5	2.139	0.0358
T300	0.125	7	39	19	−1.1	0.0	−4.81	0.72	1.273	0.0234

increments of 0.1. Another modification involved treating x_3 and x_4 as variables of the integer and discrete types, respectively. The optimum results obtained are summarized in Table 2.

The second example problem pertains to the design of a cantilever composite laminate beam. The beam and the applied, concentrated, and distributed loads are shown in Fig. 5, with details of the geometrical configuration of the multilayered laminate shown in Fig. 6. The beam structure was analyzed as a special case of a symmetric laminated composite plate; details of the analysis are available in Ref. 17. The design of the laminated composite depends on material properties, as well as geometry and configuration variables such as lamina thickness, number of plies, fiber orientation in each ply, and the volume fraction. These variables are of both the continuous and discrete types. With reference to Fig. 6, the depth of the beam can be described by three layers, with each layer containing N_1, N_2, and N_3 plies of thickness t. The fiber orientations in these layers are given by θ_1, θ_2, and θ_3, respectively. The optimization problem is formulated as a multidimensional nonlinear programming problem with continuous, discrete, and integer variables. The mathematical problem statement is as follows:

$$\text{Minimize } (F,\Delta_z)$$

where

$$F = 2t\,(N_1+N_2+N_3)\,bL\,\rho_c g \tag{22}$$

$$\Delta_z = PL^3/3E_x^b I \tag{23}$$

subject to

$$\Delta y \le L/1000, \quad \omega_1 \ge 125 \text{ Hz}, \quad \omega_2 \ge 750 \text{ Hz}, \quad H \le 2.5 \text{ cm} \tag{24a}$$

$$\left(\frac{\sigma_L}{\sigma_{LU}}\right)^2 - \left(\frac{\sigma_L}{\sigma_{LU}}\right)\left(\frac{\sigma_T}{\sigma_{LU}}\right) + \left(\frac{\sigma_T}{\sigma_{TU}}\right)^2 + \left(\frac{\tau_{LT}}{\tau_{LTU}}\right)^2 \le 1 \tag{24b}$$

Here F is the structural weight and Δ_z the tip deflection in the z direction; I is the moment of inertia relative to the midplane; and E_x^b is defined in Ref. 20 as the effective beam modulus of the laminated beam.

In the preceding formulation, the width b and length L were chosen as 2 cm and 25 cm, respectively. The density of the composite ρ_c was defined in terms of fiber density ρ_f, matrix density ρ_m, and the fiber volume fraction V_f as follows:

$$\rho_c = \rho_f V_f + \rho_m (1 - V_f) \tag{25}$$

The thickness of each ply was considered to be a discrete variable, with admissible values selected from the following set:

$$t = (0.1, 0.125, 0.15, 0.175, 0.225, 0.275, 0.325, 0.375)$$

The number of plies in the three layers were considered as integer variables and allowed to assume values between 3 and 100. The fiber orientations were allowed to vary between −90 and +90 deg, and the volume fraction bounded between 0.5 and $\pi/2\sqrt{3}$. The fiber orientations and volume fraction are variables of the continuous type. The constraints defined in Eqs. (24) include a tip displacement in the y direction, lower bounds on the first and second natural frequencies, an upper bound on the depth of the beam, and a strength constraint obtained on the basis of the Tsai-Hill failure theory.[21] In the strength constraint, σ_{LU} and σ_{TU} are the allowable tensile strengths in the longitudinal and transverse directions, respectively, and τ_{LTU} is the allowable shear strength. These allowable stress levels are listed in Ref. 17. For the optimization problem described earlier, the initial design was selected as follows:

$$t = 0.2 \text{ mm}, \quad N_1 = 5, \quad N = 30, \quad N_3 = 14$$

$$\theta_1 = 10 \text{ deg}, \quad \theta_2 = 40 \text{ deg}, \quad \theta_3 = -30 \text{ deg}, \quad V_f = 0.5$$

The ideal solution was obtained by treating all design variables as continuous and, for two representative fiber materials, was as follows:

$$f^{id} = (0.93095\ N,\ 0.01548 \text{ mm}) \quad \text{for Scotch glass}$$

$$= (0.34428\ N,\ 0.00633 \text{ mm}) \quad \text{for T300 graphite}$$

These ideal solutions were used in conjunction with the weighting coefficient technique to generate a series of Pareto optimal solutions for different weight coefficients. Table 3 shows representative results for a Scotch-epoxy composite structure for both continuous- and mixed-type design variables. Optimum design variables for Scotch and T300 fibers for the case when both criteria are equally weighted are shown in Table 4.

Concluding Remarks

The paper presents a methodology for solving multiobjective optimization problems that involve a mix of discrete, continuous, and integer design variables. The method combines a discrete variable variant of the global criterion approach with a branch-and-bound strategy. The mathematical basis for the approach is also presented. For discrete and integer design variable problems, it is shown that the ideal solution required in the global criterion strategy is one that corresponds to a continuous optimal solution. The paper has also attempted to establish the advantages of a multicriterion optimization strategy in relation to the conventional approach of using a scalar objective function and accounting for other competing criteria by design constraints. The method developed in this work was implemented on illustrative multiobjective design problems with encouraging results.

Acknowledgments

This work was supported by the U.S. Army Research Office under Grant DAAL03-88-K-0013. The first author would like to thank Professor W. Stadler of California State University at San Francisco for providing philosophical insights into the importance of the multicriterion design problem.

References

[1]Ragsdell, K. M., "The Utility of Nonlinear Programming Methods for Engineering Design," *Proceeding of the 11th ONR Naval Structural Mechanics Symposium on Optimum Structural Design*, Oct. 1981.

[2]Berke, L., Khot, N. S., "Use of Optimality Criteria Methods for Large Scale Systems," AGARD LS 70, *Structural Optimization*, Oct. 1974, pp. 1-29.

[3]Pareto, V., "Cours D'economie Politique," F. Rouge, Lausanne, 1896.

[4]Baier, H., "Uber Algorithmen zur Ermittlung und Charakterisierung Pareto-optimaler Losungen bei Entwurfsaufgaben elastischer Tragwerke," *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 57, 1977, pp. 318-320.

[5]Baier, H., "Mathematische Programmierung zur Optimierung von Tragwerken Inbesondere bei Mehrfachen Zielen," Ph.D. Thesis, Darmstadt Univ., Federal Republic of Germany, 1978.

[6]Koski, J., "Truss Optimization with Vector Criterion," Pub. 6, Tampere Univ. of Technology, Tampere, Finland, 1979.

[7]Koski, J., "Multicriteria Optimization in Structural Design," *Proceedings of the International Symposium on Optimum Structural Design*, Oct. 1981.

[8]Osyczka, A., "An Approach of Multicriterion Optimization Problems for Engineering Design," *Computer Methods in Applied Mechanics and Engineering*, Vol. 15, 1978, pp. 309-333.

[9]Osyczka, A., "An Approach to Multicriterion Optimization for Structural Design," *Proceedings of the International Symposium on Optimum Structural Design*, Oct. 1981.

[10]Rao, S. S., and Hati, S. K., "Game Theory Approach in Multicriteria Optimization of Function Generating Mechanisms," *Journal of Mechanical Design, Transactions of the American Society of Mechanical Engineers*, Vol. 101, 1979, pp. 398-405.

[11]Leitmann, G., "Some Problems of Scalar and Vector-Valued Optimization in Linear Viscoelasticity," *Journal of Optimization Theory and Applications*, Vol. 23, 1977, pp. 93-99.

[12]Hwang, C. L., and Masud, A. S. M., "Multiple Objective Decision Making: Methods and Applications; A State-of-the-Art Survey," *Lecture Notes in Economics and Mathematical Systems*, Vol. 164, Springer-Verlag, Berlin-Heidelberg-New York, 1979.

[13]Glover, F., and Sommer, D., "Pitfalls of Rounding in Discrete Management Decision Variables," *Decision Sciences*, Vol. 22, No. 4, Dec. 1975, pp. 445-460.

[14]Gomory, R. E., "An Algorithm for Integer Solutions to Linear Programs," Princeton-IBM Mathematics Research Project, TR 1, Nov. 1958.

[15]Garfinkel, R. S., and Nemhauser, G. L., *Integer Programming*, Wiley, New York, 1974.

[16]Vanderplaats, G. N., "An Efficient Feasible Direction Algorithm for Design Synthesis," *AIAA Journal*, Vol. 22, Oct. 1984, pp. 1633-1640.

[17]Hajela, P., and Shih, C.-J., "Optimal Design of Laminated Composites Using a Modified Mixed Integer and Discrete Programming Algorithm," *Computers and Structures* (to be published).

[18]Boychuk, L. M., Ovchinnikov, V. O., "Principal Methods for Solution of Multi-criteria Optimization Problems (Survey)," *Soviet Automatic Control*, Vol. 3, 1973, pp. 1-4.

[19]Salukvadze, M. E., "Vector Optimization Problems in Control Theory," Mecniereba, Tbilisi, 1975 (Russian).

[20]Whitney, J. M., *Structural Analysis of Laminated Anisotropic Plates*, Technomic, 1987.

[21]Jones, R. M., *Mechanics of Composite Materials*, McGraw-Hill, New York, 1975.